Bayesian Estimation of Shape Correspondence - Towards a Continuity Prior

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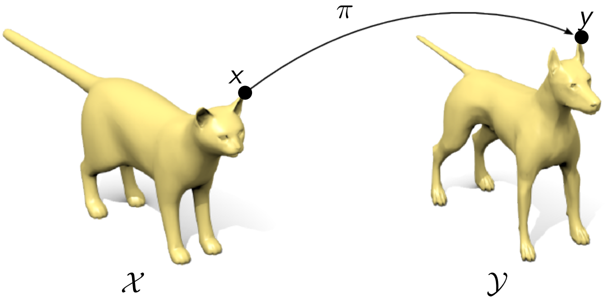
Retreat · 2016

Based on a joint work with Roee Litman, Emanuele Rodolà, Alex Bronstein and Daniel Cremers

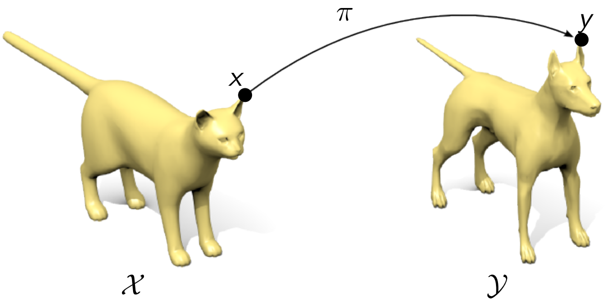
Shape correspondence



Point-wise map $\pi : \mathcal{X} \to \mathcal{Y}$

Semantically meaningful!

Shape correspondence



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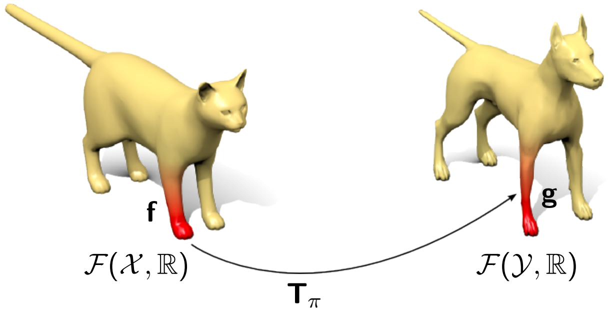
Functional Maps



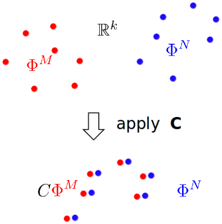
Functional maps $\mathbf{T}_\pi : \mathcal{F}(\mathcal{X}, \mathbb{R}) \rightarrow \mathcal{F}(\mathcal{Y}, \mathbb{R})$

Ovsjanikov et al., 2012

Functional maps - recovery

$$\hat{y}(x) = \underset{y \in \mathcal{Y}}{\operatorname{argmin}} \| T_F(\delta_x) - \delta_y \|_{L^2}^2$$

$$\approx \underset{y \in \mathcal{Y}}{\operatorname{argmin}} \| C \begin{pmatrix} \Phi_1^{\mathcal{X}}(x) \\ \vdots \\ \Phi_k^{\mathcal{X}}(x) \end{pmatrix} - \begin{pmatrix} \Phi_1^{\mathcal{Y}}(y) \\ \vdots \\ \Phi_k^{\mathcal{Y}}(y) \end{pmatrix} \|_2^2$$



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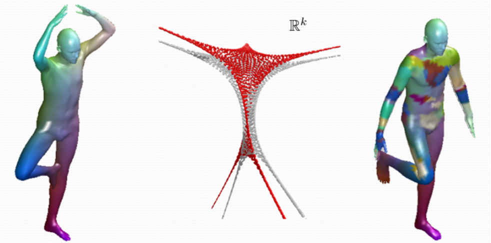
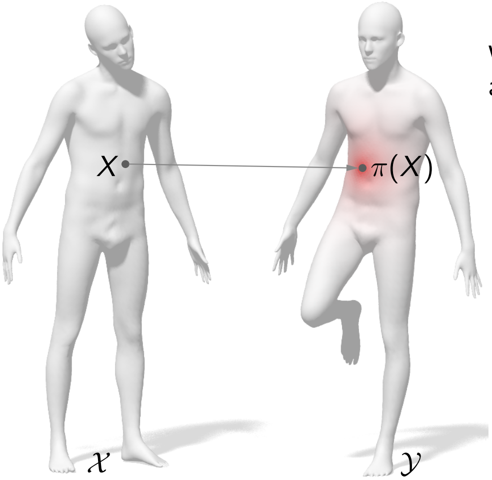


Geodesic error of recovery

Rodolà,Möller,Cremers, 2015

Probabilistic approach



We treat points on the shapes as random variables X, Y

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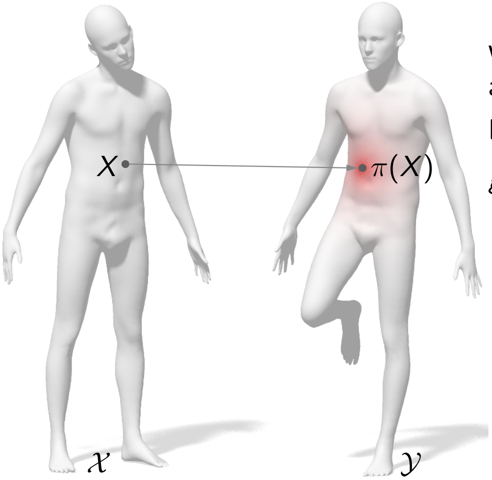

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Notation

$$g(x) = \mathbb{P}(X = x)$$

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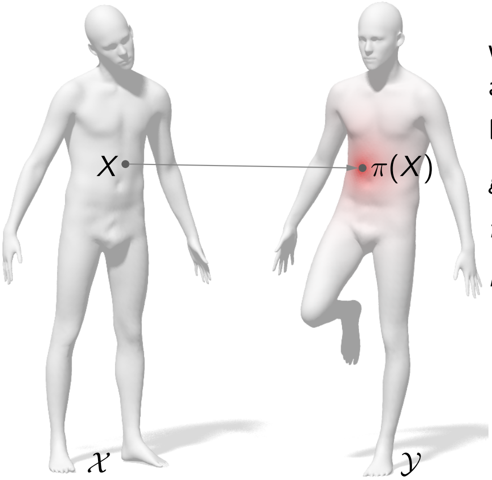

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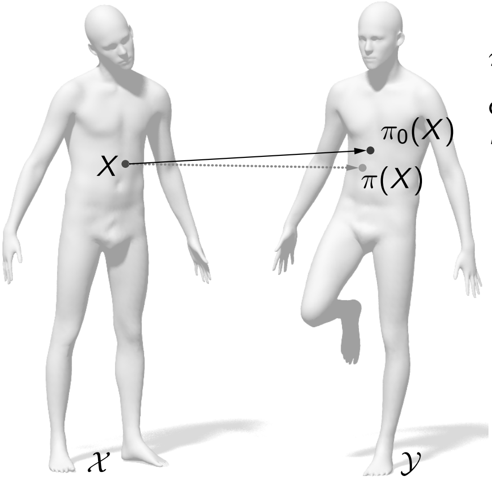
Notation

$$g(x) = \mathbb{P}(X = x)$$

$$f(y|x) = \mathbb{P}(\pi(X) = y|X = x)$$

$$h(x|y) = \mathbb{P}(\pi^{-1}(Y) = x|Y = y)$$

Probabilistic approach 2



$$\pi_0(X) = \pi(X) + N$$

coming from other method
N gaussian noise

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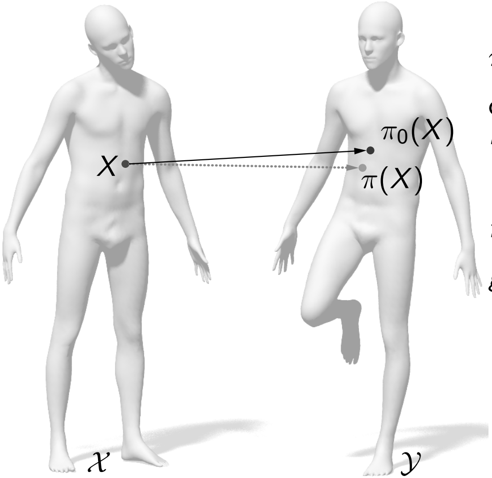

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$g(x)$ constant (uniform)

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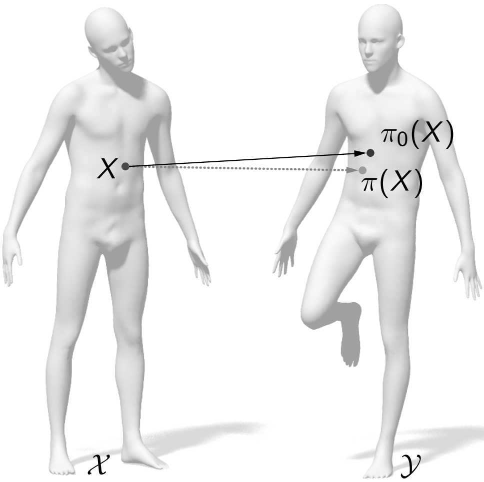

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Simple assumptions

more sophisticated
models possible

Bayesian estimation

$$h(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int_\Theta f(x|\theta)g(\theta)d\theta}$$

Let $l : \Theta \times \Theta \to \mathbb{R}$ be a loss function ($l(a, \theta)$: what does it cost if θ is guessed to be a?).

The expected loss if the parameter is chosen to be a (given the observation x) is given by

$$E(l(a, \cdot)|x) = \int_\Theta l(a, \theta)h(\theta|x)d\theta$$

The Bayesian estimator

$$\hat{\theta} = \underset{a \in \Theta}{\text{argmin}} E(l(a, \cdot)|x) = \underset{a \in \Theta}{\text{argmin}} \int_\Theta l(a, \theta)h(\theta|x)d\theta$$

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Bayesian estimation - translated to shapes

The Bayesian estimation of the preimage of $y \in \mathcal{Y}$ is given by

$$\begin{aligned}
\hat{\pi}^{-1}(y) &= \underset{x \in \mathcal{X}}{\operatorname{argmin}} E(l(x, \cdot)|y) \\
&= \underset{x \in \mathcal{X}}{\operatorname{argmin}} \int_{\mathcal{X}} l(x, x') \frac{f(y|x')g(x')}{\int_{\mathcal{X}} f(y|\tilde{x})g(\tilde{x})d\tilde{x}} dx' \\
&= \underset{x \in \mathcal{X}}{\operatorname{argmin}} \int_{\mathcal{X}} l(x, x') f(y|x') dx' \\
&= \underset{x \in \mathcal{X}}{\operatorname{argmin}} \int_{\mathcal{X}} l(x, x') \exp\left(-\frac{d_{\mathcal{Y}}^2(y, \pi_0(x'))}{2\sigma^2}\right) dx'
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Intrinsic mean and median filter

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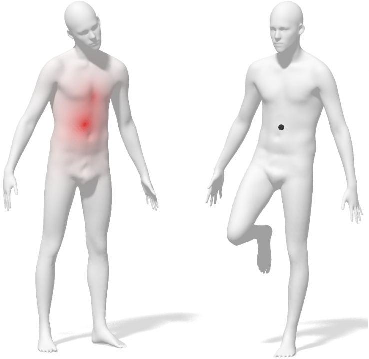

$p = 2$ intrinsic mean

$p = 1$ intrinsic median

Why intrinsic mean?

Let $\Omega \subset \mathbb{R}^n$ be a euclidean domain and $h : \Omega \to [0,1]$ a probability density and define

$$E(x) = \int_\Omega d^2(x,x')h(x')dx'$$
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$$\nabla E(x) = \int_\Omega 2xh(x')dx' - \int_\Omega 2x'h(x')dx'$$

The gradient vanishes iff

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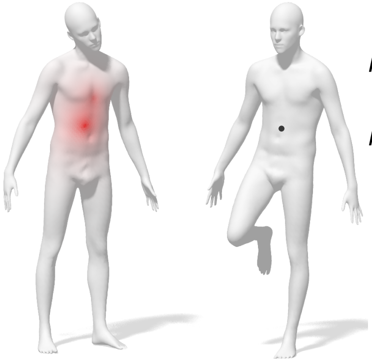

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Given a perturbed/noisy input correspondence π_0 we estimate the underlying bijective correspondence $\hat{\pi}$ via

$$\hat{\pi} = \underset{\pi: \mathcal{X} \xrightarrow{1:1} \mathcal{Y}}{\operatorname{argmin}} \int_{\mathcal{Y}} \int_{\mathcal{X}} d^p_{\mathcal{X}}(\pi^{-1}(y), x') \exp\left(-\frac{d^2_{\mathcal{Y}}(y, \pi_0(x'))}{2\sigma^2}\right) dx' \, dy$$

Discretization

Assuming $\mathcal{X}$ and $\mathcal{Y}$ discretized at n points

$$\int_{\mathcal{X}\times\mathcal{Y}} d_{\mathcal{X}}^{p}(\pi^{-1}(y), x') \exp(-\frac{d_{\mathcal{Y}}^{2}(y, \pi_0(x'))}{2\sigma^2}) dx' dy$$

$$\approx \sum_{i,j=1}^{n} (\mathbf{D}_{\mathcal{X}})^{p}_{\pi^{-1}(j),i}\, e^{-\frac{(\mathbf{D}_{\mathcal{Y}})^2_{\pi_0(i),j}}{2\sigma^2}} \mu_i \nu_j$$

$$= \operatorname{trace}\left(\mathbf{\Pi}^{\mathrm{T}} \mathbf{P} \mathbf{\Gamma}\right)$$

- $\mathbf{\Pi} = n \times n$ permutation matrix
- $(\mathbf{\Gamma})_{ij} = (\mathbf{D}_{\mathcal{X}})^{p}_{ij} \mu_i \nu_j$
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$$\min_{\Pi} \operatorname{trace}\left(\mathbf{\Pi}^{\mathrm{T}}\mathbf{P}\mathbf{\Gamma}\right)$$

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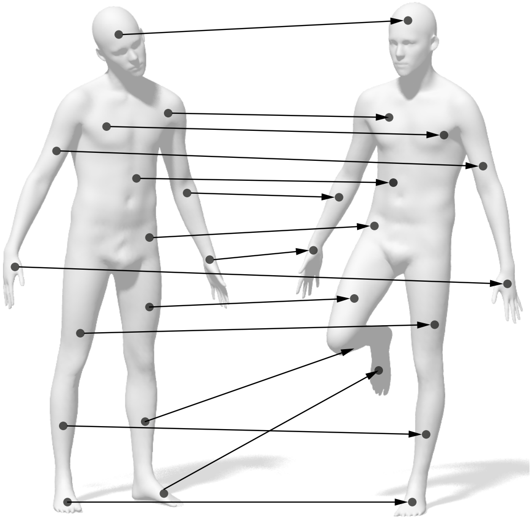
Multiscale acceleration



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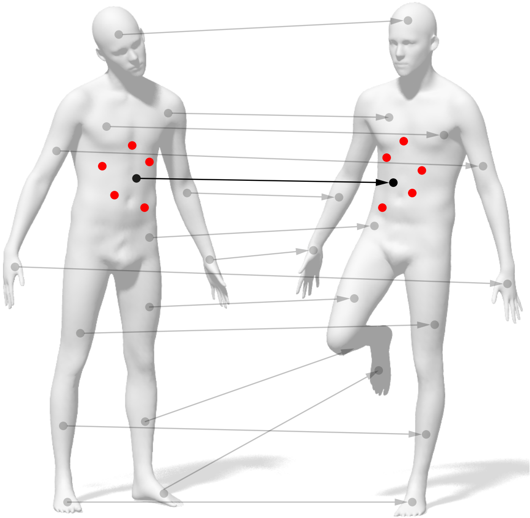

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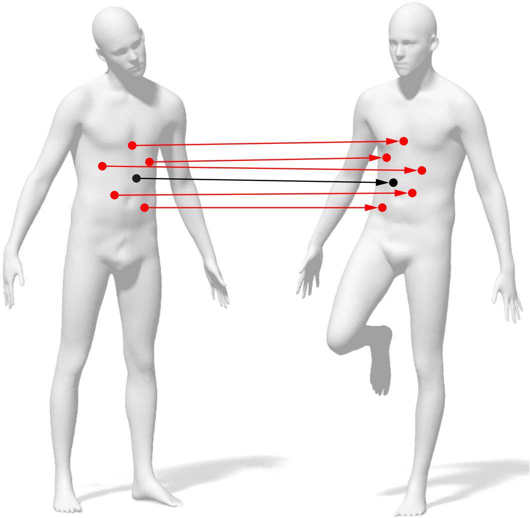

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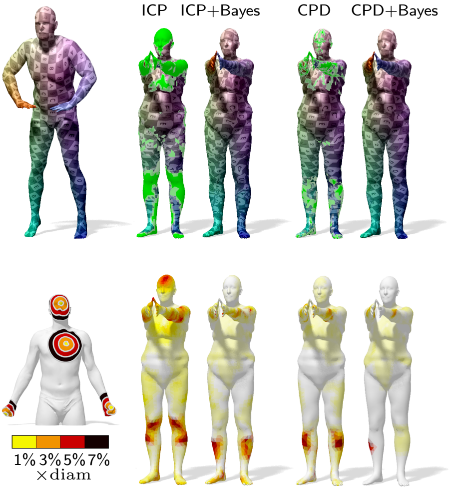

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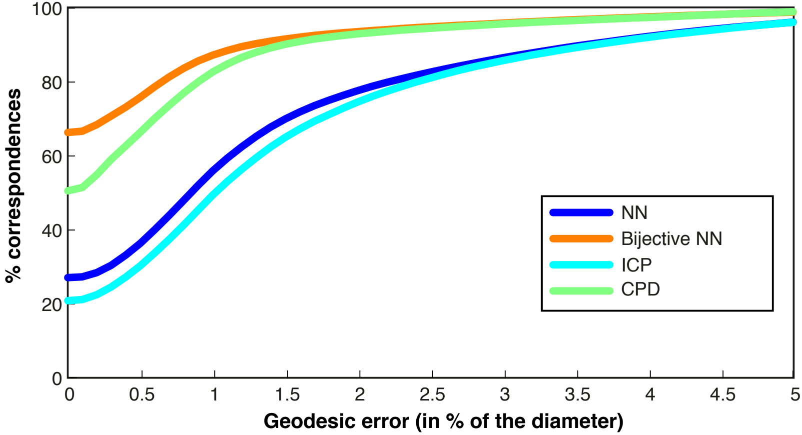
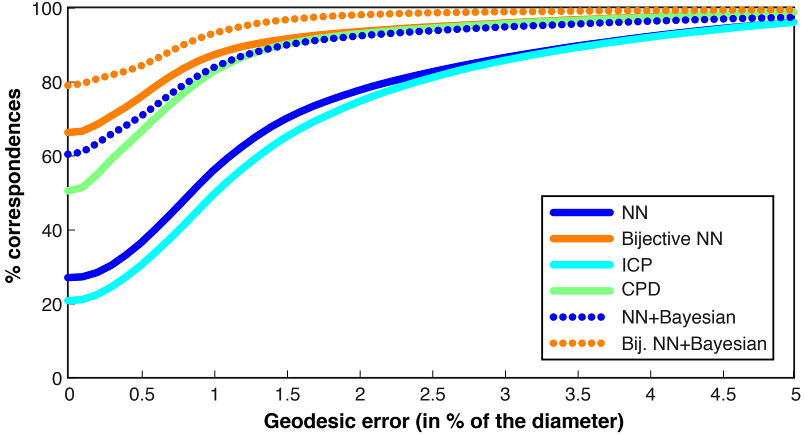


Data: FAUST; Evaluation: Kim et al., 2011; CPD: Rodolà et al., 2015

Evaluation: Functional correspondence



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Run times

n	1000	1000	6890	6890
k	20	50	20	50
Nearest neighbors	0.04	0.06	1.35	2.88
Bijective NN	2.79	2.30	463.66	253.03
ICP	0.14	0.24	12.72	30.08
CPD	4.79	4.67	1745.06	2085.65
NN + Bayesian	1.75	1.28	382.86	244.10
Bij. NN + Bayesian	4.06	3.44	746.00	440.94

Iterative process

The inference step can be iterated:

$$\pi_{k+1} = \operatorname*{argmin}_{\pi:\mathcal{X}\overset{1:1}{\rightarrow}\mathcal{Y}} \int_{\mathcal{Y}} \int_{\mathcal{X}} d_{\mathcal{X}}^{p}(\pi^{-1}(y), x') \exp\left(-\frac{d_{\mathcal{Y}}^{2}(y, \pi_k(x'))}{2\sigma^2}\right) dx'\, dy$$

Open question: Can this be seen as some type of gradient descent of a smoothness supporting energy?

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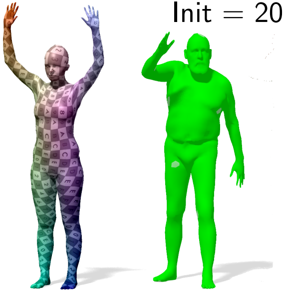
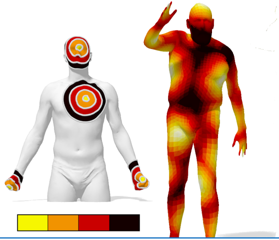
Evaluation: Sparse correspondence



Init = 20 sparse corresponding points



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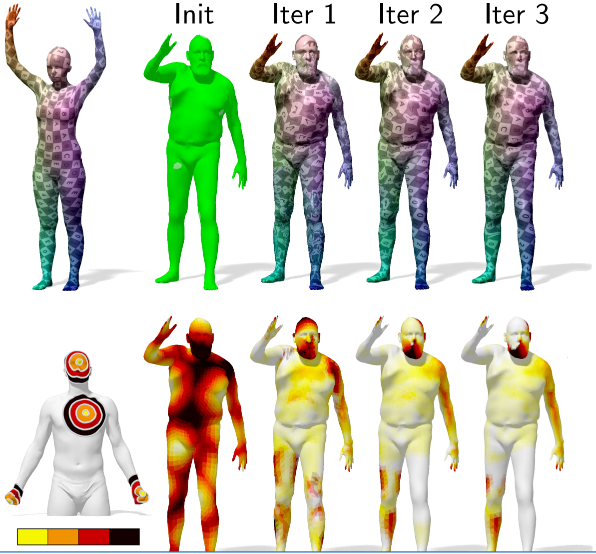
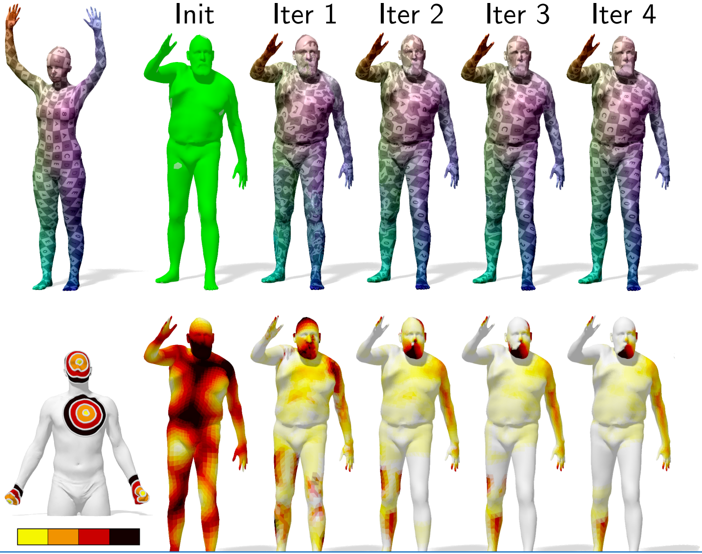
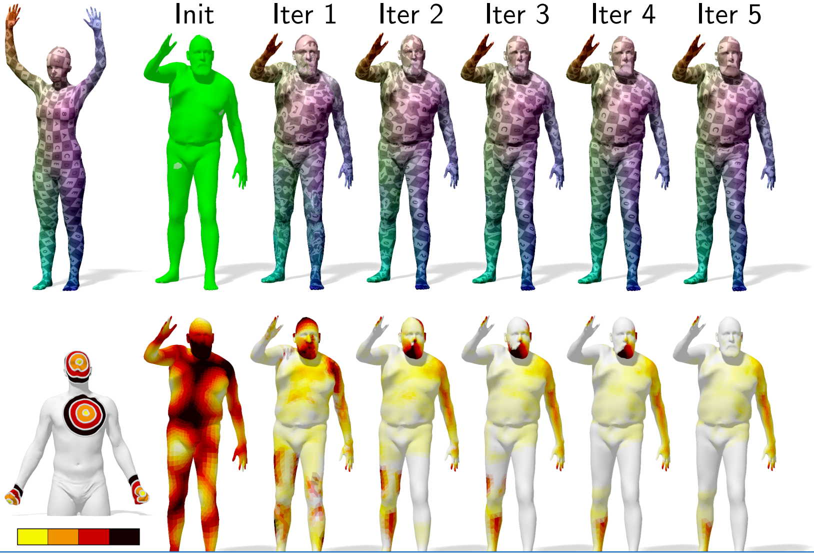


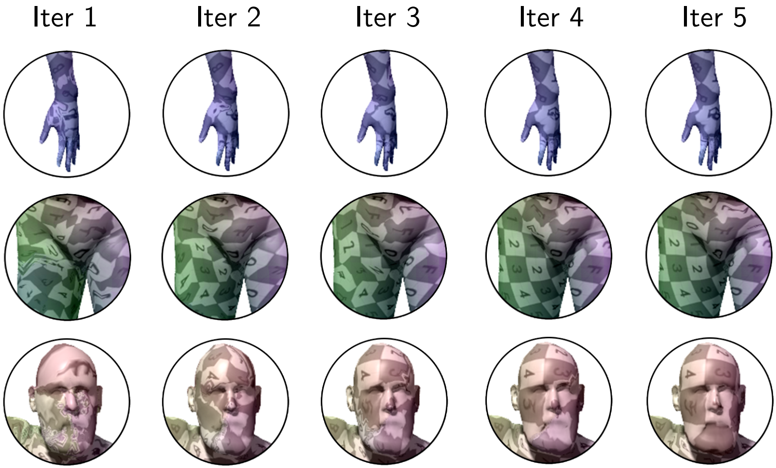
Evaluation: Sparse correspondence



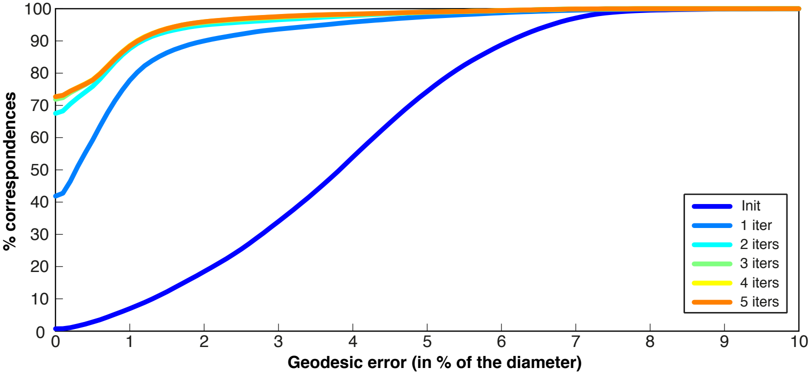
Evaluation: Sparse correspondence



Evaluation: Sparse correspondence

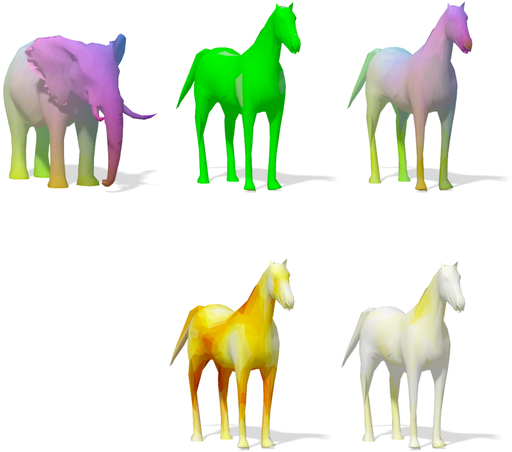


Evaluation: Sparse correspondence



Data: FAUST; Evaluation protocol: Kim et al., 2011

An elefant approach to shape matching



Conclusion

- Correspondence as an inference problem from stochastic data
- Intrinsic analogy of mean and median filter
- More interesting noise models: intrinsic analogy of bilateral or non-local means filter
- No isometry assumptions (horse/elephant)
- Towards a continuity/smoothness prior for shape matching

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Thanks for your attention